Practical 1P2

Young's Modulus and Stress Analysis

What you should learn from this practical

Science

This practical ties in with the lecture courses on elasticity. It will help you understand:

1. Hooke's law of elasticity;
2. how to relate strain states to different orientations;
3. relations between elastic moduli E, G and ν.

Practical skills

You will learn how to use strain gauges to determine elastic properties of a material. You will learn how to use spreadsheets to process data and present your results graphically.

Data analysis and experimental methods

The practical ties in with the lectures on Introduction to Errors and Measurement. You will learn how to assess, minimise, and report errors in experimental data.

Overview

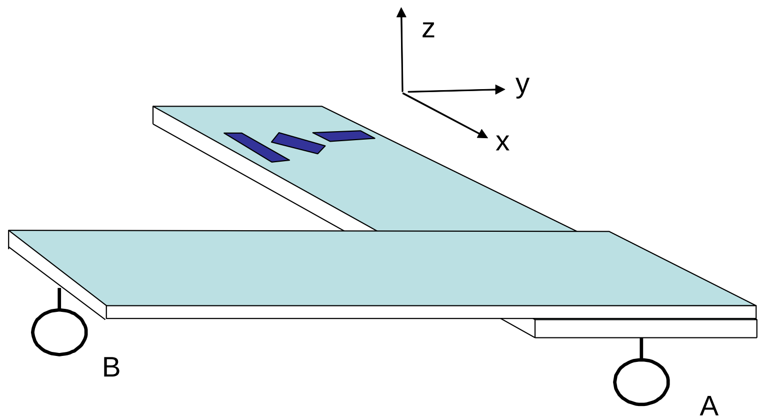
The objectives of this practical are

1. to demonstrate Hooke's law;
2. to determine the Young's modulus and Poisson's ratio of a material;
3. to determine the shear modulus of the material;
4. to check the inter-relation of E, G and ν.
5. to make reasoned estimates of experimental errors.
6. to think about how to minimise errors in performing measurements.

The device used is the electrical resistance strain gauge. This is the most widely used device for measuring elastic strains. It is essentially a strip of metal foil which is well glued to the surface where the strain is to be measured, so that when the material is strained, the strain at the surface is fully transmitted to the metal foil. Elastic strain along the length of the strip causes a small change in resistance of the gauge, largely because of the change in length and cross-sectional area of the strip, although there is also a slight change in its resistivity. Because small changes in resistance are easy to measure accurately, the gauge gives an accurate reading of the small elastic strain along the direction of the strip in the gauge. The change in resistance, and hence voltage across the strain gauge for a constant current, is proportional to the strain; the gauge manufacturer supplies the value of the constant of proportionality. For the strain gauges and current used here $\varepsilon = 3.803\times10^{-7}$ V, where ε = strain and V = voltage across strain gauge (measured in microvolts).

Experimental details

The experimental work in this practical is very simple but proper working out of the results and assessment of errors, which should be done as much as possible during the practical period, will take some time. You should do this practical in groups of two or three, but the writing up should be done as individuals.



Bending

This part of the practical involves the simple cantilever bending of the beam to which the strain gauges are attached.

Suspend the load pan from point “A”, and apply successively larger loads to the end of the beam. Record the strain gauge outputs produced in the x and y directions; also record the readings from the central angled gauge. Plot suitable graphs of the gauge readings as a function of the applied load (use a spreadsheet). The linearity of the graphs will demonstrate the validity of Hooke's law.

Convert the strain gauge readings to strains ε_x and ε_y, and the loads to Newtons. From the ε_x and ε_y readings, the loads applied and the dimensions of the beam, calculate the Young's modulus E and Poisson's ratio ν of the beam (Use the gradients of the stress/strain lines, rather than individual readings - the LINEST function in Excel is useful here.) The theory for this part of the practical is given in appendix 1.

Make a reasoned estimate of the accuracy of your values. What are the main sources of error and why? How has your experimental method contributed to the error? How has your data processing method contributed to propagation of errors into your final answer?

Based on the value for E and the estimated accuracy of your measurement, what material do you think the beam is made of?

From the ε_x, ε_y and ε_θ readings, work out the angle at which the central angled gauge is fixed, together with an estimate of the accuracy of your value. The theory for this part of the practical is given in appendix 2.

Torsion

This part of the practical involves torsion (twisting) of the beam to which the strain gauges are attached.

Use the scissor-jack table to support loading eye “A” at the end of the strain-gauged beam, so that the beam cannot deflect downwards. Apply a load to loading eye “B”, and record the strain gauge outputs produced in the x and y directions, and from the central angled gauge. Explain the result.

Now apply successively larger loads, and record the appropriate strain gauge output. Convert loads and strain gauge outputs to shear stress (see appendix 3) and shear strain. Plot the data and obtain a value of the shear modulus G = $\sigma_{xy}/2\varepsilon_{xy}$. As before, establish the accuracy of your value.

For isotropic elastic materials,

$$G = \frac{E}{2(1+\nu)}$$

Check the consistency of the values you have obtained with this equation.

Safety considerations

No unusual safety problems with this practical.

Rough timetable

Day 1:

Introduction to practical by SD.
You should be able to do most, if not all, of the experimental work

Day 2:

Finish off experiments, if needed.
Analyse results and begin write-up.

What should be in the write up

Structure of write-up

Structure the report as it is normally done in a scientific paper:
Background and aims, Experimental method, Results, Discussion, Conclusions.

Length of write up

The write-up should be about 4-5 pages maximum, plus graphs of results and diagrams.

Marking considerations

Critical assessment of primary sources of error in measurements
Establishment of the resulting errors in the quantities to be determined
Steps taken to minimise the errors in the quantities to be determined.
Reasoned explanation of the observed phenomena

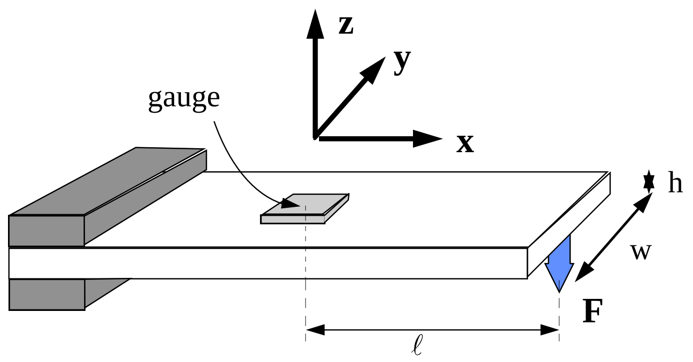
Appropriate length of write-up (i.e. not too long!).

Not wanted in write-up

More than the *brief* necessary details of the experimental procedure.

Appendices: Elasticity theory

1. Cantilever beam theory



Due to the applied force F, a couple of moment $F\ell$ (the bending moment) acts on the beam cross section at the gauge position.

The bending stress σ_x varies linearly from a maximum σ_{max} at the top surface to a minimum $-\sigma_{max}$ (compressive) at the bottom surface. Hence at a height z from the centre line of the beam, the stress σ_x is:

$$\sigma_x = \frac{\sigma_{max}\, z}{(h/2)}$$

This stress acts on an area = w.dz at a distance from the centre line (moment arm) = z. The total moments from all of these stresses internal to the beam must balance the moment $F\ell$ applied externally.

$$F\ell = \int \text{force x moment arm}$$

$$= \int \text{stress x area x moment arm}$$

$$= \int_{-h/2}^{h/2} \frac{\sigma_{max}\, z}{(h/2)} \text{w.dz}\, z$$

$$F\ell = \frac{2w\sigma_{max}}{h} \int_{-h/2}^{h/2} z^2 dz$$

$$\sigma_{max} = \frac{6F\ell}{wh^2}$$

The tensile strain ε_x at the gauge position is then

$$\varepsilon_x = \sigma_{max} / E = \frac{6F\ell}{Ewh^2}$$

The strain ε_y is given by:

$$\varepsilon_y = -\nu\varepsilon_x$$

where E is Young's modulus and ν is Poisson's ratio.

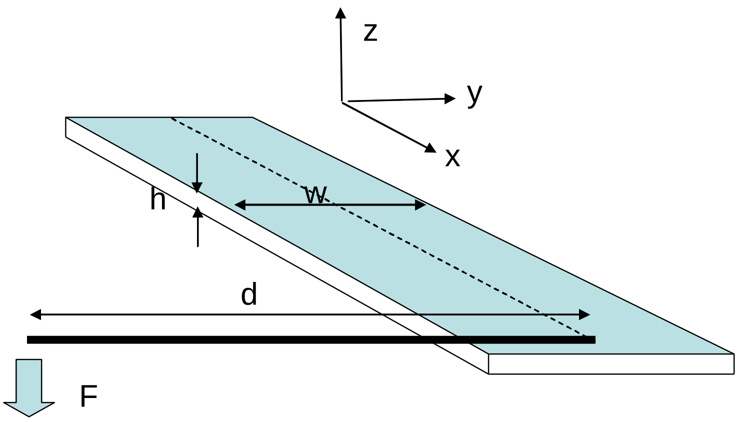
2. Strain at the central angled gauge

Stress and strain are not scalar or vector quantities. They are examples of **second rank tensors,** which describe material properties and variables that change depending on the direction one points relative to the crystal axes. You will learn more about tensor properties of materials in the second year. For the purposes of this practical it is enough to know one important result from tensor analysis, that when measuring the strain at an angle θ the result obtained is:

$$\boldsymbol{\varepsilon_\theta = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + 2\varepsilon_{xy} \sin\theta \cos\theta}$$

This equation can be used to derive θ from the readings of the three gauges, which give $\boldsymbol{\varepsilon_\theta}$, $\boldsymbol{\varepsilon_x}$ and $\boldsymbol{\varepsilon_y}$. Until you apply torsion to the beam, assume that the *shear strain* $\boldsymbol{\varepsilon_{xy} = 0}$.

3. Torsional loading



The shear stress τ at the beam surface along the dotted line is given by

$$\tau = \frac{3Fd}{wh^2}$$

Practical Questionnaire

First Year: MSOM & MEM

Term: Michaelmas

Practical no. 1P2 Young's Modulus

1) Was the practical timed correctly relative to the lectures ?
far too early -3 -2 -1 spot on 1 2 3 far too late

2) Did you think you got out of the practical what you were meant to ?
not at all 0 1 2 3 4 5 completely

3) Did you find writing up the practical:
very difficult 0 1 2 3 4 5 no problem

4) Was the practical:
completely useless 0 1 2 3 4 5 very useful

5) Was the practical:
far too short -3 -2 -1 spot on 1 2 3 far too long

6) Was the practical:
very boring 0 1 2 3 4 5 very interesting

7) Was the Junior Demonstrator
completely useless 0 1 2 3 4 5 very helpful

8) Did the practical script give you enough information to do the practical ?:
not enough info -3 -2 -1 spot on 1 2 3 too much / too confusing

Any other comments ?
(particularly for any problems exposed by responses to the questions above)

Please hand in at the same time as your report