Practical 1P3

Young's Modulus and Stress Analysis

What you should learn from this practical

Science

This practical ties in with the lecture course ***Elastic Deformation (MS2.1)***.

It will help you understand:

1. Hooke's law of elasticity
2. how to relate strain states to different orientations
3. relations between elastic moduli E, G and ν

Practical skills

You will learn how to use strain gauges to determine the elastic properties of a material. You will use spreadsheets to process data and present your results graphically.

Data analysis and experimental methods

The practical ties in with the lectures on ***Introduction to Errors and Measurement***. You will learn how to assess, minimise, and report errors in experimental data.

Assessment

For this practical your lab book will be assessed, and the mark will contribute towards Prelims.

You are also required to write a scientific report of this practical that will be assessed formatively – feedback will be given but the assessment of the report will not contribute to Prelims.

Guidance for the keeping of lab books and the writing of scientific reports is provided separately (see Canvas: ***Keeping a good lab notebook***, ***Writing a scientific report***, where you can also find the generic mark sheets).

Safety considerations

There are no unusual safety problems with this practical.

Rough timetable

Day 1:

- Introduction to practical by SD.
- Experimental work and data analysis

Day 2:

- Finish off experiments if needed, and complete data analysis.

Overview

The practical involves applying loads to a beam and measuring the resultant strains to establish the Young's modulus (E), Poisson ratio (ν), and Shear modulus (G) of the material from which the beam is made.

The objectives of this practical are

1. to demonstrate Hooke's law;
2. to determine the Young's modulus, Poisson's ratio and shear modulus of an unknown material;
3. to check the inter-relation of E, G and ν;
4. to make reasoned estimates of experimental errors;
5. to think about how to minimise errors in performing measurements.

Experimental details

The experimental work in this practical is very simple but proper working out of the results and consideration of errors will take some time.

<u>Before you start the experiment</u>, read through the whole set of instructions, and write notes in your lab book to summarise the method and the measurements that you will need to take. You are encouraged to include sketches of the experimental setup in your lab book.

Errors should be considered before and during the measurements to optimise the outcome of the measurements and provide a reliable assessment of their accuracy.

Strain Gauges

The device used to measure strain is the electrical resistance strain gauge. This is the most widely used device for measuring elastic strains. It is essentially a strip of metal foil which is well glued to the surface where the strain is to be measured, so that when the material is strained, the strain at the surface is fully transmitted to the metal foil. Normal strain along the length of the strip causes a small change in resistance of the gauge, largely because of the change in length and cross-sectional area of the strip, although there is also a slight change in its resistivity. Because small changes in resistance are easy to measure accurately, the gauge gives an accurate reading of the normal strain along the direction of the strip in the gauge.

The change in resistance, and hence voltage across the strain gauge for a constant current, is proportional to the strain. The strain can be calculated using $\varepsilon = \boldsymbol{A}\,V$, where $\varepsilon =$ strain, $V =$ voltage across strain gauge (measured in units of microvolts) and $\boldsymbol{A}$ is the calibration constant[1].

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[1] The value of the calibration constant, $\boldsymbol{A}$, to give strain when the voltage V is in μV is written on your apparatus.

Part 1: Bending

This part of the practical involves the simple cantilever bending of the beam to which the strain gauges are attached, achieved by suspending the load pan from point “A” shown in Figure 1.

The theory for this part of the practical is given in Appendix 1.

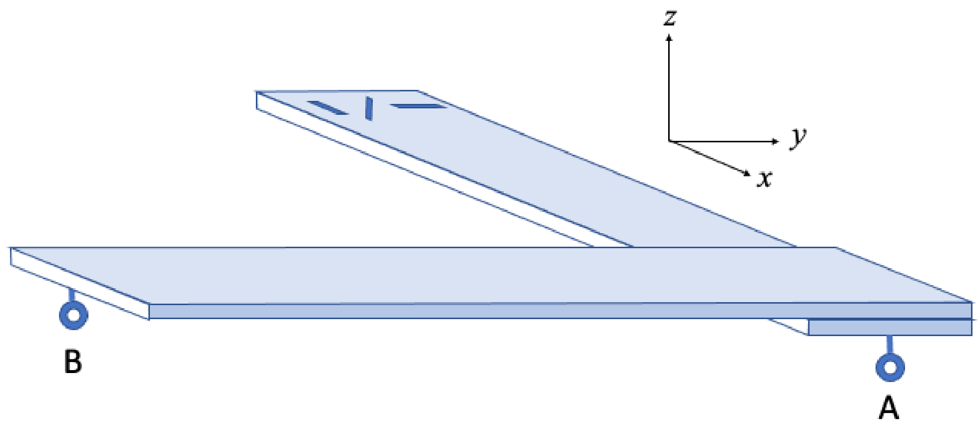


Figure 1: The cantilever beam with strain gauges, used for bending and torsion experiments

Measure the relevant dimensions of the beam and record these *in your lab book*.

Record the six strain gauge outputs *in your lab book* as you apply a range of different masses using the weights provided. The outputs correspond to the gauges pointing in the x, y and ‘angled’ directions and mounted to the top and bottom surfaces of the beam.

Plot suitable graphs of the gauge readings as a function of the applied mass (use a spreadsheet or graph paper and stick graphs into your lab book). The linearity of the graphs will demonstrate the validity of Hooke's law.

Convert the strain gauge readings of voltage to strains and convert the applied mass to the load in Newtons.

Calculate the bending stress at the beam surface and use your stress/strain data to calculate the Young's modulus E and Poisson's ratio ν of the beam (the LINEST function in Excel is useful here to find the gradients).

Use your values of Young's modulus E and Poisson's ratio ν, with the following equation for isotropic elastic materials, to determine the shear modulus G.

$$G = \frac{E}{2(1+\nu)}$$

Use your data from the strain gauges to calculate the angle θ at which each of the central angled strain gauges is fixed. The theory for this part of the practical is given in Appendix 2.

Estimate the likely errors in each measured (or stated) quantity and combine them appropriately to find the overall errors in your measurements of Young’s Modulus, Poisson’s ratio, Shear Modulus, and the angles of the central strain gauges.

Comment in your lab book on the following:

- What are the main sources of error and why?
- What type of error are they (random or systematic)?
- How has your experimental method contributed to minimising the error? (such as the number and sequence of measurements, care taken to ensure accurate readings)
- Are there obvious steps you could have taken to further reduce errors?

Part 2: Torsion

This part of the practical involves torsion (twisting) of the beam to which the strain gauges are attached. It is achieved by supporting loading eye “A” shown in Figure 1 using the scissor-jack table to prevent deflection and applying a load to loading eye “B”.

The theory for this part of the practical is in Appendix 3.

Repeat the process you carried out for the bending experiments, applying a range of masses to the beam. Record the strain gauge outputs in your lab book. Plot suitable graphs in your lab book and describe the qualitative behaviour that you observe. How do you explain it?

Use the strain gauge outputs and the torsional loads to find both the shear strain ε_{xy} and the shear stress σ_{xy} at the surface.

Plot the shear stress and shear strains and obtain a value of the shear modulus G using the following equation.

$$G = {}^{\sigma_{xy}}\!/_{2\varepsilon_{xy}}$$

Establish the error in your value and comment in your lab book on the principal sources of this error.

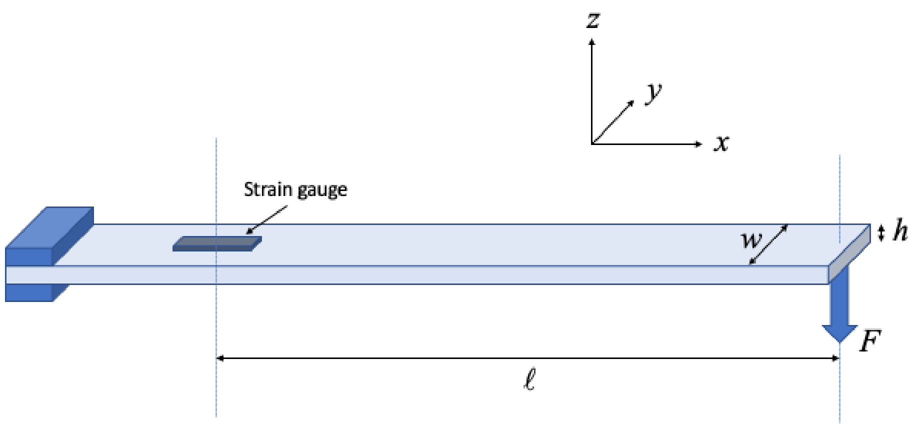
Conclusion

Conclude your lab book report with a summary of all the values measured (i.e., E, ν, G and θ), the accuracy with which they can be measured using this apparatus and the most important considerations when performing the measurements to achieve best results.

Considering the measurement errors, compare the value for G you measured by torsional loading with the value that you obtained using your measurements of E and ν in bending. If the values are inconsistent with each other, write comments in your lab book on why you think this is.

Write in your lab book what material you think the beam is made of, based on the values you have measured and their estimated accuracy.

Appendices: Elasticity theory
1. Cantilever beam theory



Due to the applied force F, a couple of moment $F\ell$ (the bending moment) acts on the beam cross section at the gauge position to cause pure bending.

There is no stress σ_y, in the y-direction, and the beam bends in a circular arc.

The bending stress σ_x varies linearly from a maximum σ_{max} at the top surface to a minimum $-\sigma_{max}$ (compressive) at the bottom surface.

Hence at a distance z from the centre line of the beam, the stress σ_x is:

$$\sigma_x = \frac{z\,\sigma_{max}}{\left(h/2\right)}$$

The stress σ_x acts on an area, $dA = w\,dz$, to give a *force* at a distance z (the *moment arm*) from the centre line, which produces a *moment*.

The total moment from all the stresses internal to the beam must balance the moment $F\ell$ that is applied externally.

$$F\ell = \int \text{force} \times \text{moment arm}$$

$$= \int \text{stress} \times \text{area} \times \text{moment arm}$$

$$= \int_{-h/2}^{h/2} \frac{z\,\sigma_{max}}{\left(h/2\right)} w\,dz\,z$$

$$F\ell = \frac{2w\sigma_{max}}{h}\int_{-h/_2}^{h/_2} z^2\,dz$$

$$\sigma_{max} = \frac{6F\ell}{wh^2}$$

The tensile strain ε_x at the surface is then:

$$\varepsilon_x = \frac{\sigma_{max}}{E} = \frac{6F\ell}{Ewh^2}$$

The strain ε_y at the surface is given by:

$$\varepsilon_y = -\nu\varepsilon_x$$

where E is Young's modulus and ν is Poisson's ratio.

2. Strain at the central angled gauge

Stress and strain are not scalar or vector quantities. They are examples of **second rank tensors,** which describe material properties and variables that change depending on the direction.

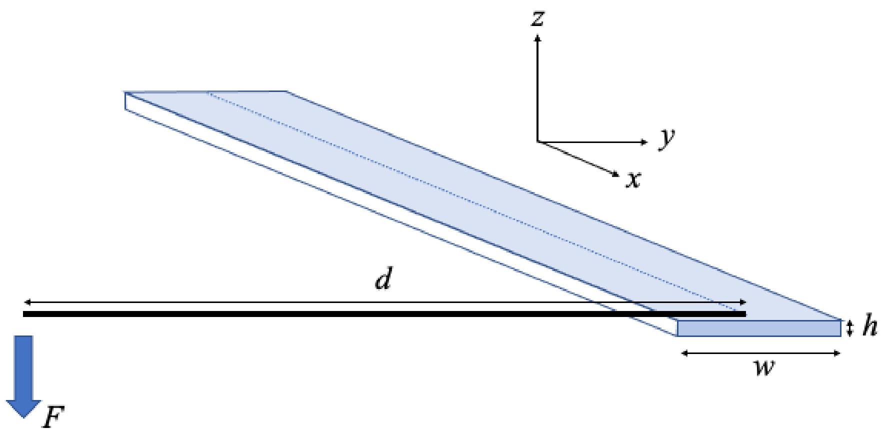
You will learn more about tensor properties of materials in the second year. For the purposes of this practical it is enough to know one important result from tensor analysis, that when measuring the strain ε_θ at an angle θ (measured in the anticlockwise direction from x to y) the result obtained is:

$$\varepsilon_\theta = \varepsilon_x\cos^2\theta + \varepsilon_y\sin^2\theta + 2\varepsilon_{xy}\sin\theta\cos\theta$$

This equation, which you will encounter in the first year as **Mohr's circle**, can be used to derive θ from the readings of the three strain gauges.

When you only apply bending to the beam, you can assume that the shear strain ε_{xy} is zero, and that the longitudinal strain ε_x and lateral strain ε_y are the same at the positions of all three strain gauges.

3. Torsional loading



The shear stress σ_{xy} (or τ) at the beam surface is given by the following equation:

$$\tau = \frac{3Fd}{wh^2}$$

To find the shear strain ε_{xy} on the surface, you may assume that the longitudinal strain ε_x and lateral strain ε_y are the same at the positions of all three strain gauges. You will also need the orientation θ of the central angled strain gauge, where the measured strain is ε_θ, and use the Mohr's circle relationship.